A term structure model with useful factors: assessing the impact of ECB's unconventional policies from 2014 to 2020

Johannes Kramer[†], Ken Nyholm[†*], Vahe Sahakyan[‡]

[†]European Central Bank
[‡]Bank for International Settlements

11 January, 2022

Abstract

A discrete-time no-dominance term structure model is developed. Four factors are modelled and explicitly defined to be: the short rate, the slope and curvature of the term structure of term-premia, and a measure that captures sovereign-specific features. We argue that these factors are directly useful for policy analysis and for investment professionals. An empirical application of the model is used to evaluate impacts on German and Italian yields of six ECB policy interventions from 2014 to 2020. We find that monetary policy actions during this period have compressed German yields by approximately 25, 85, and 125 basis points, at the 1Y, 5Y and 10Y maturities, respectively, and that these reductions exclusively result from changed to the term-premium component. In Italy the corresponding yield compressions are 55, 135, and 180 basis points, with the term premium component being responsible for approximately two-thirds of the action, while the remaining one-third stems from reductions to credit risk.

Acknowledgments: For helpful comments and suggestions we thank, without implicating, Luca Bortolussi, Fernando Monar, and Thomas Werner.

JEL classifications: C58, E43, E52, E58, G12

Keywords: Term structure of interest rates, no-dominance, sovereign specific risks, term premia, central bank asset purchases, European Central Bank.

All opinions and views expressed in this paper are those of the authors, and nothing displayed in the paper are necessarily endorsed by our employers the European Central Bank and the Bank for International Settlements.

[*]Corresponding author: Ken.Nyholm@ecb.europa.eu

1 Introduction

We develop a four factor term structure model where the underlying factors are explicitly defined as the short rate, the slope and curvature of the term structure of term premia, and a measure that accounts for long-term nominal growth expectations, country credit risk, liquidity risk, and redomination risk. The model falls in the family of tractable no-dominance models as suggested by Feunou, Fontaine, Le, and Lundblad (2021). It is straight forward to derive an arbitrage-free variant of the model[1], however we judge that, from a practitioner's perspective, there is more merit in having a model that can be estimated and extended easily, while still provides a high level of internal rigour and consistency, than a model that is arbitrage-free.[2]

As in the early fixed-income literature, e.g. Vasicek (1977) and Cox, Ingersoll, and Ross (1985), a natural starting point is to model the short rate. Although main-stream multi-factor term structure models include the level of the yield curve, or empirical approximations thereof, as a factor, see among many others Diebold and Li (2006) and Joslin, Singleton, and Zhu (2011), we include the short rate. Traditional central bank policies aim to steer the short rate and short rate expectations, and the short rate therefore represents the first link in the transmission of monetary policy to the economy. As such, it is one of the most important policy variables. In addition, to support policy and investment analysis, where various possible future short-rate trajectories need to be evaluated, it is natural to include the short rate directly as a factor, as this allows for easy generation and evaluation of such scenarios.

Existing multi-factor term structure models leave the convergence point of the rate expectations curve[3] to be specified by data. This object, i.e. the short-rate convergence point, is the backbone for the determination of the model-based term structure of term premia, as these are defined by wedge between the expectations curve and the model-fitted approximation to the observed yields. It is therefore effectively the mean of the short rate calculated from the chosen data sample that determines the level of the estimated term premia. While this assumption, that is made implicitly in any practical application involving traditional multi-factor models, may be acceptable for some model uses, it is not idea in a policy context, as economic meaning eventually will be attached to such estimated levels. For this reason, we explicitly include the long-term attraction point of the expectations curve in our model, and call it C^*, which is akin to a long-term nominal version of the r^* concept suggested by Holston, Laubach, and Williams (2017). In addition to being a long-term gauge, C^* deviates from r^* by including inflation expectations and compensation for the credit, liquidity, and redenomination risk that market participants attach to the sovereign, who issues the bonds that form the basis for the analysis.

The short rate and $C*$ factors together span the expectations-component curve. What remains for our model to explain, is therefore the wedge between the observed yields across maturities, and the expectations curve, i.e. the term-structure of term premia. We include two factors to this end: the slope and the curvature of the term-structure of term premia.

Central bank asset purchases in the fixed income markets (QE) have allegedly strong impacts on sovereign yields. Purchases have been carried out in Japan since 2001, in US since 2008, and in the Euro area since the early 2010s. Large amounts of fixed income instruments have been bought under the QE umbrella. In the case of the ECB, as of October 2021, purchases amount to Eur 3,091 billion[4]. It is therefore a key element for any economically founded yield curve model to be able to capture the impact of such large scale interventions. And, this is naturally no only relevant while purchases are on-going, it is equally important when purchases are scaled back and eventually stopped. As one transmission channel of QE is found empirically to act through term-premia[5], it appears reasonable to rely on a model with explicitly defined term-premia factors embedded in its factor structure. We use our model to assess the relative importance of the credit, liquidity, and redenomination risk channel, one the one hand, and the term premium channel, on the other hand. This is done by evaluating six ECB policy initiatives put in place from 2014 to 2020. The policy impacts are measured on the German and Italian sovereign yield curves. In the German market we find that ECB policies are exclusively transmitted via the term premium channel, i.e. as removal of duration risk. A total yield compression of 25, 85, and 125 basis points is estimated, at the 1Y, 5Y and 10Y maturities. Our model indicates that two-thirds of the overall yield compression in

[1] See Appendix A.

[2] In practice, the dividing line between the no-dominance and no-arbitrage frameworks consists in a maturity dependent constant that in empirical applications is found to be statistically equal to zero (see Coroneo, Nyholm, and Vidova-Koleva (2008)).

[3] This element is defined as $\frac{1}{n}\sum_{j=0}^{n-1} r_{t,t+j}$, and is also referred to as the $\mathbb{P}$ curve in the context of the no-arbitrage modelling framework.

[4] See: https://www.ecb.europa.eu/mopo/implement/app/html/index.en.html.

[5] See, for example, Li and Wei (2013) and Eser, Lemke, Nyholm, Radde, and Vladu (2019)

the Italian market stems from term-premium compressions, and that the remaining one-third results from compressions to the credit, liquidity, and redenomination risk component. Total compressions in Italian yields amount to 55, 135, and 180 basis points at the 1Y, 5Y and 10Y maturities.

2 The Model

Our modelling approach is based on a modification of the well-known and widely use dynamic Nelson-Siegel type of term structure models (see e.g., Diebold and Li (2006), Diebold, Rudebusch, and Aruoba (2006), and Diebold and Rudebusch (2013)). Although this class of term structure models are not strictly arbitrage-free, Feunou, Fontaine, Le, and Lundblad (2021) shows that this class of models precludes dominant trading strategies, a requirement that comes very close to fulfilling the no-arbitrage constraints. This framework ensures that portfolios with identical prices have identical expected pay-offs. Accordingly, it is not possible to generate a guaranteed positive pay-off from a long-short portfolio in the cash asset, i.e. bonds, with no initial cost. To exemplify, in a no-dominance market it is ensured that: $\alpha \cdot P_t(n_j) + (1-\alpha) \cdot P_t(n_k) = P_t(n_z),\ \forall\ \{j,k,z\} > 0$ *iff* $\alpha \cdot n_j + (1-\alpha) \cdot n_k = n_z$, where n is the maturity of a bond with price P. This criterion is less strict than the no-arbitrage criterion that to date permeates the term structure literature (see, among many others, Joslin, Singleton, and Zhu (2011), Christensen, Diebold, and Rudebusch (2011), Piazzesi (2010), Dai and Singleton (2000), Duffie and Kan (1996), and Collin-Dufresne, Goldstein, and Jones (2008)). No-arbitrage excludes all dominant trading strategies and, in addition, the strategies that offer the holder the *possibility* of making a positive pay-off, while guaranteeing that no loss will be encountered, and that no initial outlay is required. Hence, it is this latter set of strategies that distinguishes no-arbitrage from no-dominance pricing. What does this set of strategies look like? Since they afford the holder a profit-opportunity with loss-protection, it appears that derivatives must be included. The generic portfolio that encapsulates the no-arbitrage concept is therefore a bond (the underlying, s), put and call options (p, c), and a cash position equal to the present-value of the exercise $pv(X)$. And, to exclude arbitrage-opportunities the put-call parity must hold: $s + p = c + pv(X)$, while according to the no-dominance framework it is (only) guaranteed that $s + p \leq c + pv(X)$.

It is naturally an empirical question whether the set of strategies that divide no-dominance from no-arbitrage is empty, and thus whether the two principles are identical from a yield-curve practitioner's perspective. Since the model we develop is applied to euro area yields, in particular to German and Italian, yield curves, and since these markets are generally believed to be well functioning, we are comfortable with the choice of a model being developed according to the no-dominance principle.[6]

2.1 Bond prices and yields

Following the no-dominance principle suggested by Feunou, Fontaine, Le, and Lundblad (2021) for the pricing of credit-risk free bonds, the price of a 0-maturity bond is equal to 1, and recursive equations specify the pricing for bonds with residual maturities $n > 0$:

$$P_0(X_t) \equiv 1 \tag{1}$$

$$P_n(X_t) = P_{n-1}\left(g(X_t)\right) \cdot exp(-\rho' \cdot X_t) \tag{2}$$

where P denotes the price, n is the residual maturity, t counts calendar time, and the vector X holds the yield curve factors. The function $g(\cdot)$ moves the factors forward in the time-dimension, and the product $\rho'(X)$ specifies the discount rate as a function of the yield curve factors. This can be seen from the prices from the expressions of a 1-period and a 2-period bond:

$$P_1(X_t) = P_0\left(g(X_t)\right) \cdot exp(-\rho' \cdot X_t) = 1 \cdot exp(-\rho' \cdot X_t), \tag{3}$$

$$P_2(X_t) = P_1\left(g(X_t)\right) \cdot exp(-\rho' \cdot X_t). \tag{4}$$

Given that the modelled bonds are assumed to be credit-risk free, i.e. $P_0(X_t) \equiv 1$, it is seen that (3) simply amounts to the one-period discount-factor. In (2) we need an expression for the price of a bond that is one period closer to maturity on the RHS compared to the LHS, i.e. we need $P_{t+1}(n-1)$. Consequently, $g(\cdot)$, is an operator that moves its argument forward in time.[7]

[6] See Appendix A for a version of the model that fulfills the no-arbitrage principle.

[7] Drawing a comparison between the no-dominance framework, and the no-arbitrage set-up, it is noted that (2) represents $\mathbb{Q}$-measure discounting, as no adjustment is made for the market-price of risk. Consequently, $g(\cdot)$, gives the $\mathbb{Q}$-corresponding dynamics of the yield curve factors.

Through successive substitution, the price of a n-maturity bond is:

$$P_n(X_t) = exp\left(-\sum_{j=0}^{n-1} \rho' \cdot \left(\underbrace{(g \circ g \circ \cdots \circ g)}_{j}(X_t)\right)\right) \tag{5}$$

where $g \circ g$ denotes the composition of the g-function. To ensure that bond prices satisfy the properties of: i) positivity, ii) invertibility and iii) discounting distant cash flows, the following condition is adopted:

$$g(X_t) = \Phi \cdot X_t. \tag{6}$$

Accordingly, the expression for n-maturity yield is:

$$y_t(n) \equiv -\frac{\log\left(P_n(X_t)\right)}{n} = \frac{1}{n}\sum_{j=0}^{n-1} \rho' \left(\Phi^j\right) \cdot X_t \tag{7}$$

With Φ defined as:

$$\Phi \equiv \begin{bmatrix} 1-a & a & 1-\gamma & 1-\gamma \\ 0 & 1 & 1-\gamma & 1-\gamma \\ 0 & 0 & \gamma & \gamma-1 \\ 0 & 0 & 0 & \gamma \end{bmatrix} \tag{8}$$

And, ρ defined as:

$$\rho = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

It is seen that:

$$\begin{aligned} y_t(n) &= \frac{1}{n}\sum_{j=0}^{n-1} \rho' \cdot \Phi^j \cdot X_t = \rho' \cdot \sum_{j=0}^{n-1} \frac{\Phi^j}{n} \cdot X_t \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1-(1-a)^n}{an} & 1-\frac{1-(1-a)^n}{an} & 1-\frac{1-\gamma^n}{n(1-\gamma)} & \frac{1-\gamma^n}{n(1-\gamma)} - \gamma^{n-1} \\ 0 & 1 & 1-\frac{1-\gamma^n}{n(1-\gamma)} & \frac{1-\gamma^n}{n(1-\gamma)} - \gamma^{n-1} \\ 0 & 0 & \frac{1-\gamma^n}{n(1-\gamma)} & \gamma^{n-1} - \frac{1-\gamma^n}{n(1-\gamma)} \\ 0 & 0 & 0 & \frac{1-\gamma^n}{n(1-\gamma)} \end{bmatrix} \cdot X_t \\ &= \left[\frac{1-(1-a)^n}{an},\ 1-\frac{1-(1-a)^n}{an},\ 1-\frac{1-\gamma^n}{n(1-\gamma)},\ \frac{1-\gamma^n}{n(1-\gamma)} - \gamma^{n-1}\right] \cdot X_t. \end{aligned} \tag{10}$$

The yield equation of our model as portrayed in (10) can then be written as:

$$y_t(n) = b_n \cdot X_t + e_t, \tag{11}$$

where the loading structure is defined by:

$$b_n = \left[\frac{1-(1-a)^n}{an},\ 1-\frac{1-(1-a)^n}{an},\ 1-\frac{1-\gamma^n}{n(1-\gamma)},\ \frac{1-\gamma^n}{n(1-\gamma)} - \gamma^{n-1}\right]. \tag{12}$$

2.2 Interpretation of the yield curve factors

The loading structure, b_n, of our model is defined in (12), and the corresponding factor interpretations are given by:

$$X_t = \begin{bmatrix} r_t \\ C_t^* \\ \theta_t^s \\ \theta_t^c \end{bmatrix} = \left\{\begin{array}{l} \text{short rate} \\ \text{nominal long-term natural rate of interest incl. credit-risk} \\ \text{slope of term structure of term premium} \\ \text{curvature of term structure of term premium} \end{array}\right\} \tag{13}$$

b_n	Operation		
	$\lim_{n\to 0}$	$\lim_{n\to\infty}$	$\frac{\partial}{\partial n}$
$\frac{1-(1-a)^n}{an}$	$-\frac{\ln(1-a)}{a}$	0	$\frac{(1-a)^n-1}{a\,n^2} - \frac{\ln(1-a)\,(1-a)^n}{a\,n} < 0$
$1-\frac{1-(1-a)^n}{an}$	$\frac{\ln(1-a)}{a}+1$	1	$\frac{\ln(1-a)\,(1-a)^n}{a\,n} - \frac{(1-a)^n-1}{a\,n^2} > 0$
$1-\frac{1-\gamma^n}{n(1-\gamma)}$	$1-\frac{\ln(g)}{g-1}$	1	$-\frac{g^n\,n\,\ln(g)-g^n+1}{n^2\,(g-1)} > 0$
$\frac{1-\gamma^n}{n(1-\gamma)}-\gamma^{n-1}$	$\frac{\ln(g)}{g-1}-\frac{1}{g}$	0	$\frac{g^n\,n\,\ln(g)-g^n+1}{n^2\,(g-1)} - \frac{g^n\,\ln(g)}{g} \gtreqless 0$

Note: The table provides insights on the key properties of the factor-loadings derived in (12), by showing the limits for $n \to 0$ and $n \to \infty$ and the first derivatives. The parameters a and g determine the value loadings as $n \to 0$. As $n \to \infty$ the loadings converge to 0 or 1. The sign and functional form of the first derivative show that the loadings for factors 1-3 are monotonously increasing or decreasing functions. The derivative of loading 4 shows that this loading reaches a maximum point somewhere in the maturity spectrum.

Table 1: Factor loading properties

To demonstrate the validity of (13) we rely on Table 1 and Figure 1. The yield-curve loading for factor 1, $b_n(1,1)$, is monotonously decreasing from its starting point at 1, observed at the shortest maturity covered by the data sample used to estimate the model, and approaching 0 as the maturity increases. Together with the definition of ρ, and the observation that the other three factors all have loadings equal to 0 at the shortest maturity observation shows, that the first factor is equal to the short rate. Since $b_n(1,1)+b_n(1,2)=1$ and the limit of $b_n(1,2)$ is 1 as the maturity increases, we can see that the second factor is the long-term convergence point of the short rate process. Hence, the second factor is C^*, and the two first factors together with their loadings trace out the duration risk-free (drf) curve (also referred to as the expectations curve):[8]

$$\begin{aligned} y_t^{drf}(n) &\equiv \frac{1}{n}\cdot \mathbb{E}_t \sum_{j=0}^{n-1} r_{t,t+j} \\ &= \frac{1-(1-a)^n}{an}\cdot r_t + 1 - \frac{1-(1-a)^n}{an}\cdot C^*. \end{aligned} \tag{14}$$

The wedge between the expectations curve and the model-fitted curve is then, by definition, filled by the remaining two yield curve factors and their loadings, $b_n(1,3:4)$:

$$\begin{aligned} y_t(n) &\equiv y_t^{drf}(n) + \theta_t(n) \\ &= y_t^{drf}(n) + 1 - \frac{1-\gamma^n}{n(1-\gamma)}\cdot\theta_t^s + \frac{1-\gamma^n}{n(1-\gamma)} - \gamma^{n-1}\cdot\theta_t^c + e_t, \end{aligned} \tag{15}$$

where e_t is the residual between the yield curve fitted as the sum of the duration risk free curve and the term structure of term premia, as defined by the second and third terms in (15).

[Figure 1 around here]

2.3 Factor dynamics

Within the no-dominance framework there is no explicitly hard-wired relationship between the parameters that govern the cross-sectional behaviour of yields and those that govern the time series behaviour of the factors. This is in contrast to the no-arbitrage framework (see, e.g. Joslin, Singleton, and Zhu (2011)) where the market price of risk serves this role. This is fully in line with the traditional agnostic approach of the Dynamic Nelson-Siegel model (see, e.g. Diebold and Li (2006)). Our yield curve factors are therefore assumed to be appropriately modelled in the context of a VAR(1) model:

$$X_t = k + K\cdot(X_{t-1}-k) + v_t \tag{16}$$

[8] In the context of deriving the yield in the cross-sectional dimension, i.e. for pricing purposes, it is noted that our model implies an AR(1) process for the short rate: $r_t = \kappa\cdot C^* + (1-\kappa)\cdot r_{t-1} = c + \beta\cdot r_{t-1}$, where $c=\kappa\cdot C^*$, and $\beta = 1-\kappa$.

where k is the vector of means, K is the matrix of autoregressive coefficients and v is the residuals. X hold the yield curve factors described above.

To cater for the unspanned factor, C_t^*, the VAR model is adapted accordingly:

$$\begin{bmatrix} X_t \\ C_t^* \end{bmatrix} = \begin{bmatrix} k_X \\ k_{C^*} \end{bmatrix} + \begin{bmatrix} K_{X,X} & K_{X,C^*} \\ K_{C^*,X} & K_{C^*,C^*} \end{bmatrix} \cdot \left(\begin{bmatrix} X_{t-1} \\ C_{t-1}^* \end{bmatrix} - \begin{bmatrix} k_X \\ k_{C^*} \end{bmatrix} \right) + \begin{bmatrix} v_{X,t} \\ v_{C*,t} \end{bmatrix} \tag{17}$$

To populate C^* we adapt the concept of the long-term long-maturity equilibrium rate, R^*, see Roberts (2018), encompassing inflation expectations and sovereign credit risk. $\bar{R}^*$ is similar in spirit to r^* suggested by Holston, Laubach, and Williams (2017), but it it is a long-maturity concept, e.g. a 10-year rate. In our context, which is based on nominal yields, we need to add inflation expectations, and since there may also be credit risk on the sovereigns, we also include this possibility.

Specifically, we assume that:

$$C^* = \bar{R}^* + E[\pi] + E[s(PD)], \tag{18}$$

where π is the inflation rate and $s(PD)$ is a fair compensation for expected default expressed as a yield spread.[9]

Furthermore, we also include the amount of free-floating government bonds, denoted by FF, i.e. the amount of bonds that are available to private investors in the market. The free-float variable captures the impact of central bank QE purchases by adjusting the sovereign debt outstanding by holdings of foreign official institutions and the domestic central bank. Recent literature[10] include the impact of the free-float on the term-structure of term premia. However, in the context of a monetary union, it is possible that QE also impacts the credit risk that market participants assign to individual sovereigns, see Costain, Nuño, and Thomas (2021). Therefore, we allow for two ways that the free-float can impact the yield curve: via the traditional term-premium channel and via a credit risk channel, see Equation (23).

Expanding the new Keynesian approach suggested by Roberts (2018) with the free-float variable, $\bar{R}^*$ is defined by:

$$R_t^* = \bar{R}_t^* + cyc_t \tag{19}$$
$$\bar{R}_t^* = \bar{R}_{t-1}^* + \alpha_1 \cdot FF_t + \epsilon_t \tag{20}$$
$$cyc_t = \alpha_2 \cdot cyc_{t-1} + \eta_t \tag{21}$$

where,

$$R_t^* = R_t^{real} + \frac{1}{\sigma} \cdot (xGap_t - \alpha_3 \cdot xGap_{t-1}) + \zeta_t \tag{22}$$
$$s\left(PD_t\right) = s\left(PD_{t-1}\right) + \alpha_4 \cdot FF_t + \omega_t \tag{23}$$

Equation (19) shows that $\bar{R}$ captures the long-term trend in the equilibrium long-maturity rate, as the cyclical component, cyc, is modelled directly, and Equation (23) shows the assumed evolution of the sovereign probability of default. Appendix B further illustrates the approach for obtaining monthly estimates of the output gap $xGap$ that are consistent with annual ones from the IMF World Economic Outlook.

3 Empirical Results

3.1 Model Estimation

The model is fitted to German and Italian zero-coupon yields observed monthly, covering the period from January 2003 to September 2021, and observed at maturities, $n = \{3, 12, 24, 36, 48, 60, 84, 120\}$-months. A central part of the empirical application of the yield curve model is its dependence on the amount of freely traded government bonds. To capture this aspect, we define the free float in the following way:

$$FF = \frac{\texttt{Gov. debt} - \texttt{holdings of foreign official inst.} - \texttt{Holdings of domestic CB}}{\texttt{Gov. debt}} \tag{24}$$

[9] We rely on Moody's (2010) to calculate this metric.

[10] See, for example, Vayanos and Vila (2009), Joyce, Miles, Scott, and Vayanos (2012), Li and Wei (2013) Eser, Lemke, Nyholm, Radde, and Vladu (2019).

Country	3m	12m	24m	36m	48m	60m	84m	120m
Germany	4.6	6.4	3.7	1.5	2.8	3.0	2.6	3.6
Italy	4.9	7.7	4.5	2.6	4.0	4.3	3.7	5.1

Note: The table shows the obtained Root Mean Squared Errors (RMSEs) from fitting the suggested model to German and Italian zero-coupon yields. RMSEs are shown for the each maturity at which yields are observed, from 3-month to 10-years. $\texttt{RMSE} = \left[\frac{1}{T} \cdot \sum_{t=1}^{T} \left(y_t(n) - \hat{y}_t(n)\right)^2\right]^{\frac{1}{2}}$, with T being the number of observations in the sample.

Table 2: Root Mean Squared Errors in basis points

where "Gov.", "inst.", "CB", stand for "Government", "institution", and "central bank", respectively. IMF data from Arslanalp and Tsuda (2012) are used to derive FF.

Table 2 reports the obtained Root Mean Squared Errors (RMSEs) in basis points (bps). A close fit is obtained with RMSEs for all maturities of 5 bps or less. Figure 2 compares the observed and fitted yields for the 3m, 3Y, and 10Y maturities.

[Figure 2 around here]

3.2 Scenario projections

Using the factor structure of the suggested term structure model we generate conditional projections for the term-premium slope and C^*, on the basis of the evolution of the Free float variable around relevant monetary policy dates. Scenarios are generated under the assumption that the free-float variable remains constant at the value observed on the given date, for all subsequent dates. Impacts of the implemented policies are then evaluated as the difference between the scenario and the in-sample fitted values for y_t^{drf} and the term structure of term premia at selected maturities, for a time-period of six months.

More precisely, we start by generating scenario projections for the selected horizon, $\tau = 6m$:

$$\begin{bmatrix} \widetilde{C}^*_{t=t^\dagger \to t^\dagger+\tau} \\ \widetilde{\theta}^s_{t=t^\dagger \to t^\dagger+\tau} \end{bmatrix} = E\left[\begin{matrix} C^*_{t=t^\dagger \to t^\dagger+\tau} \\ \theta^s_{t=t^\dagger \to t^\dagger+\tau} \end{matrix}\middle| FF_{t=t^\dagger \to t^\dagger+\tau} = FF_t^\dagger; \left\{\hat{r}, \hat{\theta}^c\right\}_{t=t^\dagger \to t^\dagger+\tau}, \hat{a}, \hat{g}, \hat{k}, \hat{K}\right] \tag{25}$$

This is done using the Kalman-filter with the estimated model parameters and assuming that the short rate and the curvature of the term structure of term premia are known with certainty. Applying the estimated loadings in (12), the observed factors and the scenario projections, we calculate the scenario projections for the duration risk-free term structure and the term structure of term premia as:

$$\widetilde{y}^{drf}(n)_{t^\dagger \to t^\dagger+\tau} = b_n[\,:, 1:2\,] \cdot \left[r, \widetilde{C}^*, \widetilde{\theta}^s, \theta^c\right]_{t^\dagger \to t^\dagger+\tau} \tag{26}$$

$$\widetilde{TP}(n)_{t^\dagger \to t^\dagger+\tau} = b_n[\,:, 3:4\,] \cdot \left[r, \widetilde{C}^*, \widetilde{\theta}^s, \theta^c\right]_{t^\dagger \to t^\dagger+\tau} \tag{27}$$

where $b_n[\,:, x:y\,]$ refers to columns $\{x, y\}$ in the loading matrix.

Scenario impacts are calculated relative to the evolution of the yield curve constituents in (15) fitted to the whole data sample:

$$\overline{y}^{drf}(n)_{t^\dagger \to t^\dagger+\tau} = y^{drf}(n)_{t^\dagger \to t^\dagger+\tau} - \widetilde{y}^{drf}(n)_{t^\dagger \to t^\dagger+\tau} \tag{28}$$

$$\overline{TP}(n)_{t^\dagger \to t^\dagger+\tau}(n) = TP(n)_{t^\dagger \to t^\dagger+\tau} - \widetilde{TP}(n)_{t^\dagger \to t^\dagger+\tau} \tag{29}$$

Six policy announcement dates are selected where the ECB introduced major new initiatives. These dates are shown in Figure 3 together with the evolution of the free float variables recorded for Germany and Italy. For each selected date we apply the framework outlined above to estimate the policy impact on the duration risk free term structure y^{drf} and on the term structure of term premia. Results are presented as the cumulative policy impact in basis points, during a period from the end of the month prior to the announcement till six months after the respective announcement. Table 3 gives an overview of the chosen policy events.

[Figure 3 around here]

Event	Event date	Evaluation start date
Negative deposit rate and more	5-Jun-2014	31-May-2014
PSPP is initiated (Eur 60 bn/month)	22-Jan-2015	31-Dec-2015
PSPP is increased (Eur 80 bn/month)	15-Mar-2016	29-Feb-2016
PSPP resumes purchases	12-Sep-2019	31-Aug-2019
PEPP announcement (Eur 750 bn)	18-Mar-2020	29-Feb-2020
PEPP extension by (Eur 600 bn)	4-Jun-2020	31-May-2020

Note: The table shows the evaluated policy events. Event, announcement date, and evaluation date are shown. PSPP is the Public Sector Purchase Programme, and PEPP is the Pandemic Emergency Purchase Programme.

Table 3: Policy evaluation dates

3.3 Results

June 2014

To revive euro zone inflation that had fallen almost linearly since 2012, and which had remained below 1% since Q3:2013, see Figure 4, the ECB launched several significant policy initiatives on 5 June 2014:

- a new targeted programme that provided Eur 400 bn of cheap credit to eurozone banks to lend to small firms;
- preparatory work was intensified on developing a new market for banks to bundle together (securitise) loans made to small companies;
- all the eurozone interest rates were cut: the headline rate was lowered to 0.15%, and the rate for bank deposits was lowered to -0.1%.
- liquidity would be injected into the financial system by no longer 'sterilising' sovereign bonds bought during the crisis in 2011 and 2012 (under the securities market programme);
- during the press conference Draghi further iterated that: *"We think what we have presented a significant package. Are we finished? The answer is no. We are not finished here. If need be, within our mandate, we are not finished here."*, and "*If required, we will act swiftly with further monetary policy easing. The Governing Council is unanimous in its commitment to using also unconventional instruments within its mandate should it become necessary to further address risks of too prolonged a period of low inflation.*"

The joint impact of the announced policies are displayed in Figure 5. It is seen that the announced measures left a significant imprint on German and Italian yields. A compression of around 45 bps was observed in Germany, over a six month horizon. All of the compression in the German market came via the term premium channel, as the duration risk-free curve was left unaffected. Italian yields also experienced an overall compression of around 45 bps - but 30 bps of the overall compression came from the term premium channel, while the remaining 15 bps came from a fall in the duration risk-free curve.[11]

Although there is no direct QE announcement at the meeting, it is clearly communicated by the ECB president during the press conference that it may come soon and that the Governing Council is unanimous in its assessment. Figure 3 shows that the amount of free-float in both Germany and Italy decreases slightly after the June meeting. The decrease in Germany started before the meeting and in Italy the free-float started to decrease after then meeting, admittedly at a very slow pace.

Decreases of the term premia components in Germany and Italy are expected reactions following previous empirical results assessing the impact of central bank QE programmes in UK, US, and Europe. For example, Joyce, Lasaosa, Stevens, and Tong (2011) find that the Bank of England's quantitative easing

[11] Recall that our yields curve model separates the overall impact of monetary policy into country specific effects on the term structure of term premia, and on the duration risk-free curve (also denoted by the expectations component). We can therefore isolate the monetary policy imprint on observed yields from (a) credit risk, liquidity risk, and redenomination risk extraction, via the expectations component, and (b) duration risk extraction via the term structure of term premia, see equation (13).

policy through February 2010 compressed medium to long-term term-premia in the UK gilt market by about 100 basis points; Li and Wei (2013) find that the Federal Reserve's first and second large-scale asset purchase programs, and the maturity extension program, jointly reduced the ten-year Treasury term premium by about 100 basis points; Eser, Lemke, Nyholm, Radde, and Vladu (2019) find a similar sized term premium compression in the euro area as the impact of the extension of the ECB's purchase programme extension in June 2018.

In contrast to previous studies, we also report a significant impact from central bank policies on the duration risk-free curve. It is observed that 33% of the recorded decrease in the Italian yields comes from the 15 bps fall in y^{drf}. This decrease can be attributed to a reduction in credit risk, liquidity risk, and redomination risk. The information content in our data does not allow for a decomposition of this decrease into its constituent elements. Increases in the y^{drf} curves, in both Germany and Italy - but not necessarily in equal amounts - would have been realised if market participants had priced the impact of the ECB announcements to reflect an improvement of the economic growth outlook. With the benefit of hindsight, it is clear that the situation at the time, did not justify the pricing of positive growth expectations. Rather, as shown in Table 3 the measures announced in June 2014 were only the first in a sequence of additional ECB policy interventions.

[Figure 4 around here]

[Figure 5 around here]

January 2015

January 2015 marks the start of the ECB's government bond asset purchases (PSPP). Against the backdrop of weak inflation readings the following policy initiatives were announced at the meeting on 22 January:

- starting in March the Eurosystem would purchase euro-denominated investment-grade securities issued by euro area governments and agencies and European institutions in the secondary market. Monthly purchases of Eur 60 billion would be carried out and were intended to last until end-September 2016.
- the pricing of the remaining targeted longer-term refinancing operations would be changed such that the applicable interest rate was set equal to the rate on the Eurosystem's main refinancing operations (MRO), thereby removing the 10 basis point spread over the MRO rate that applied to the first two operations.

An asymmetric impact is seen across the German and Italian market following ECB's introduction of public sector asset purchases, with term premia compression in the German market and compression of the duration risk-free curve in the Italian market. The German duration risk-free curve is unmoved by the introduced policy, and remains so throughout the 6-months evaluation window. German term premia is compressed by around 7 bps at the 1-year segment, 25 bps at the 5-year segment, and by 35 bps at the 10-year segment. It is thus the medium to long term maturity segments that are most impacted. While there is a mixed and minor impact on Italy term premia, a substantial and immediate impact is observed on the the Italian duration free curve. Although the impact is largest in the 10-year segment, the main reaction to the policies are seen in the short to medium term maturities, with a reduction of 10 bps in the 1-year segment and around 20 bps in the 5-year segment. The maximum compression of the Italian yields is seen at the 10-year segment amounting to just under 25 bps.

Given these impacts it is likely that the introduction of the PSPP led to the removal of duration risk in the German fixed income market, as illustrated by the exclusive reduction in term premia, while credit, liquidity, and redomination risk was extracted from the Italian market, as indicated by the predominant reduction in the duration risk free curve, y^{drf}.

[Figure 6 around here]

March 2016

At the Governing council meeting on 10 March 2016 it was decided to increase the monthly purchase envelope of the PSPP from Eur 60 billion to Eur 80 billion. This was not the only policy that was introduced at the meeting. More specifically, the following were decided:

- the interest rate on the main refinancing operations of the Eurosystem would be decreased by 5 basis points to 0.00%; the marginal lending facility rate would be decreased by 5 basis points to 0.25%; and the deposit facility would be decreased by 10 basis points to -0.40%.
- as mentioned, the monthly purchases under the asset purchase programme were increased from Eur 60 billion to Eur 80 billion.
- investment grade euro-denominated bonds issued by non-bank corporations established in the euro area would be included in the list of assets that are eligible for regular purchases.
- a new series of four targeted longer-term refinancing operations (TLTRO II), each with a maturity of four years, would be launched. Borrowing conditions in these operations could be as low as the interest rate on the deposit facility.

Expanding the purchase envelope of the PSPP from Eur 60 billion to Eur 80 billion, in conjucture with the other significant policy initiatives announced, led to duration extraction extraction in the German market in an amount of approximately 40 bps at the 10-year maturity point. As in the policy assessments above, the duration extraction is strongest for medium to long term segments, with the 1-year term premium being reduced by around 10 bps. There is no extraction of credit, liquidity, and redenomination risk in the German market as y^{drf} is unaffected by the policy announcements. In the Italian market the overall yield compression amounts to 20-30 bps, with the majority of the compression coming form duration extraction, of around 15-20 bps, and the remaining coming from the elements embedded in y^{drf}, i.e. credit, liquidity, and redenomination risk.

[Figure 7 around here]

September 2019

The ECB had formally stopped asset purchases, apart from reinvestments, on 13 December 2018. At the meeting on 12 September 2019 purchases were restarted along a series of other initiatives:

- monthly purchases under the asset purchase programme would be resumed at a monthly pace Eur 20 billion, and reinvestments of maturing principal amounts would continue in full;
- the possibility of buying assets with yields below the interest rate on the deposit facility was extended to all parts of the APP;
- a two-tier system was introduced for reserve remuneration where institutions' holdings of excess liquidity would be exempt from the negative deposit facility rate;
- the maturity was increased from two to three years and a voluntary repayment option was introduced for the third series of targeted longer-term refinancing operations.

Restarting PSPP impacted German term premia and led to a decrease concentrated in the medium to long-term segments, with a decrease of 15 bps at the 10-year segment, 10 bps at the 5-year segment, and less than 5 bps in the 1-year segment. In Italy the reintroduction of purchases had largest impact on term premia, and thus mainly removed duration risk, while extraction of credit, liquidity, and redenomination risk amounted only to around 10 bps. Duration extraction from the Italian bond market is estimated to be around 40 bps at the 10-year segment, 30 bps at the 5-year segment, and 10 bps at the 1-year segment. Again, the majority of the duration extraction takes place at the medium to long-term maturities.

The somewhat muted impact recorded in the German market may indicate that the potential for further duration compression in that market is limited, and that the series of ECB interventions and policy initiatives have exercised significant impacts on the German yield curve.

[Figure 8 around here]

March and June 2020

As a response to the mounting adverse financial and economic developments following the outbreak of the global coronavirus pandemic and to safeguard the monetary policy transmission mechanism, the ECB announced three initiatives at its policy meeting on 18 March:

- A new temporary pandemic emergency purchase programme (PEPP) was launched with an initial envelope of Eur 750 billion of private and public sector securities. It was originally planned that purchases would end in 2020. In June 2020 the purchase envelope was increased by Eur 600 billion, and an additional increase of Eur 500 billion was announced in December 2020, leaving the total PEPP envelope at Eur 1,850 billion and the expected end to the programme was set to 2023.
- Corporate sector purchases (CSPP) were expanded to cover non-financial commercial paper, making all commercial paper of sufficient credit quality eligible for purchase under the CSPP.
- Collateral standards were adjusted by including claims related to the financing of the corporate sector.

Alongside a globally decorating macroeconomic outlook the financial markets in the euro area were put under pressure showing larger losses than in most other jurisdictions. For example equity markets were showing 1-month losses of nearly 40%, while equities in the US, Japan and emerging markets had fallen by around 25%. Implied volatility had trended upwards and reached levels as seen during the global financial crisis in 2008. Expectations of increased fiscal spending by government, to stem adverse impacts of the pandemic, would inadvertently lead to a greater supply of government bonds, and would therefore counteract the impact of the PSPP.

Consequently, the PEPP was initiated to remedy potential disruptions to the monetary transmission mechanism by increasing the scope for risk absorption from the financial markets via additional bond purchases. Figure 9 shows the initial impacts of the March 2020 announcement on the duration free yield curve and on term premia in the German and Italian bond markets.

[Figure 9 around here]

The PEPP announcement in March 2020 left no impact on the the German duration risk-free curve, while the Italian y^{drf} curve experienced a very small and transitory increase of 3-8 bps across the shown 1 to 10 year maturities. A marginal reduction was seen in the German term premia of around 7 bps at the 10-year maturity with smaller impacts at lower maturities. The largest impacts were observed in Italian term premia, where increases of around 5, 20, and 30 bps were seen for the 1, 5, and 10-year maturities, respectively. Increasing term premia is clearly contrary to what ECB's intention was with the PEPP.

Term premia increases, as seen in Italy across all maturities in March 2020, could indicate that financial markets were underwhelmed by the initial communicated modalities of the PEPP. In March 2020 the ECB had already grown its balance sheet by Eur 2,300 billion of asset purchases, and in a environment of elevated COVID-19 uncertainty, the communicated envelope of Eur 750 billion, and the time-limited scope of the purchases were perhaps deemed to be to little to bring about the intended effects, as illustrated by the Italian term premia impact-estimates seen above. In its 29-30 April 2020 meeting the ECB conveyed the views that: (a) "*[...] The rapidly evolving situation needed to be reflected in an adjustment to the Governing Council's monetary policy stance. Both the tail risks associated with the present macro-financial crisis and its scale were assessed to have deteriorated substantially since the monetary policy meetings on 11-12 March and on 18 March*", and (b) that "*[...] most euro area sovereign bond spreads had widened relative to German benchmark bonds, irrespective of their credit rating. In some jurisdictions, bond spreads had temporarily returned to, or even risen above, the levels observed before the announcement of the pandemic emergency purchase programme (PEPP), compounded by expectations of increased issuance needs and the high prevailing uncertainty surrounding the economic and financial fallout from the crisis.*".[12] Taken together, this indicates that ECB assesses that the initial PEPP envelope may have been calibrated to a COVID-shock that was smaller in magnitude, than what it eventually turned out to be.

The assessment portrayed in the referred ECB accounts is also captured by the evolution of the 10-year yields during the first and second PEPP events. Figure 10 shows that the evaluation date for the initial PEPP announcement is followed by an increase in the Italian yield which reaches a maximum of 3% at the

[12] See the accounts of the ECB meeting on 29-30 April 2020, https://www.ecb.europa.eu/press/accounts/2020/ html/ecb.mg200522 f0355619ae.en.html.

end of March 2020. After an initial decrease, that still left the 10-year yield at levels higher than those seen at the end of February, the 10-year yield increases during the months of April and May, and had started to fall prior to the extension of the PEPP that occurred in June 2020.

[Figure 10 around here]

German yields were left untouched by the PEPP extension in June 2020, apart from a negligible transitory increase of less than 5 bps. In contrast, both the duration risk-free curve, y^{drf}, and term premia decreased substantially by 20 bps at the 10-year segment. A fast and uniform compression is seen in y^{drf} by 8, 18, and 23 bps at the 1, 5, and 10-year maturities. The compression in term premia is slower but is roughly of the same size as the compression seen in the duration risk-free curve.

[Figure 11 around here]

Summary of the impacts

Sums of the estimated impacts are calculated across countries, maturities, and evaluation horizons, and the results are shown in Figure 12 and Table. As the impact estimated for the March 2020 event, when PEPP was first introduced, is probably shrouded by excessive noise given the proximity to the Covid-19 outbreak, we calculate and show the sums of the impact over the five ECB policy events on 5 June 2014, 22 January 2015, 15 March 2016, 12 September 2019, and 4 June 2020.

[Figure 12 around here]

It is clear that the analysed policy initiatives undertaken by the ECB are transmitted to the German yield curve via the temp premium channel. The Cumulative impact measured at the 10 year segment for the credit, liquidity, and redenomination risk channel, 6 months after policy initiatives, is estimated to be less than 0.5 bps (see Table 4 at the 10Y maturity and the 6 months evaluation horizon, the impact on the C^* curve is estimated to be -0.41 basis points). Hence, all implemented policies influence German yields only as reductions to the term premium component, i.e. as reductions in the duration risk available to investors in the market. This transmission channel is inline with the theoretical underpinnings presented by Vayanos and Vila (2009) that forms the basis for most of the empirical literature in this area.

The compression observed in the Italian term premium component is overall very similar to the compressions seen in the German market. Comparable reductions are observed across the two markets when looking at the term premium impacts for each included maturities, i.e. when comparing the sum of the reductions in Germany and Italy and at the 1Y, 5Y, and 10Y maturity-columns. Although impacts are not necessarily identical across each of the analysed policy dates, in turns out that, in total, similar patterns and amounts of duration extraction has taken place in German and Italian fixed income markets during the main ECB policy interventions during 2014 to 2020. Comparing the summary results shown in Table 4 and Figure 12 on the one hand, and the duration extraction displayed in Figures 5 - 11, one the other hand, it appears that the pace of duration extraction is more pronounced in Germany during the period from 2014 to 2016, than it was in italy, however, for the policy interventions in the remaining part of the analysed period, this picture changed and the duration extraction was strongest in the Italian markets. One explanation for this is possibly that the potential for duration extraction in Germany was exhausted first, as a consequence of the higher fraction of international investors in this market compared to the Italian market.

While the policy interventions has left no imprint on the German duration risk-free yield curve, in aggregate, one-third of the risk extraction in the Italian market stems from reductions in the credit, liquidity, and redenomination risk components (y^{drf}). However, the risk reduction (measured in basis points) depends on the evaluation horizon and the maturity of the yield curve. For example, at the 3-month evaluation horizon, there is twice as much credit, liquidity and redenomination risk extracted, at the 1Y maturity segment, than there is duration risk extracted (compare the 27 bps y^{drf} decrease with the corresponding 15 bps reduction in the 1Y term premium); At the 6-month evaluation horizon, still at the 1Y maturity segment, the magnitude of duration extraction has increased and surpasses slightly the reduction in y^{drf} (compare -24 bps with -32 bps, respectively). This stronger and faster impact on credit, liquidity, and redenomination risk is seen across all tested maturity segments in the Italian market.

	Germany					
	y^{drf}			TP curve		
Horizon / Maturity	1Y	5Y	10Y	1Y	5Y	10Y
0	0.00	0.00	0.00	0	0	0
1	0.00	-0.01	-0.01	-5	-16	-24
2	-0.01	-0.02	-0.02	-12	-34	-51
3	-0.02	-0.05	-0.07	-16	-47	-70
4	-0.05	-0.11	-0.14	-21	-61	-91
5	-0.09	-0.20	-0.25	-24	-71	-106
6	-0.14	-0.33	-0.41	-29	-86	-128
	Italy					
	y^{drf}			TP curve		
Horizon / Maturity	1Y	5Y	10Y	1Y	5Y	10Y
0	0	0	0	0	0	0
1	-15	-31	-37	-3	-7	-10
2	-23	-48	-56	-11	-30	-42
3	-27	-55	-65	-16	-43	-61
4	-26	-53	-63	-21	-58	-81
5	-24	-51	-59	-25	-69	-97
6	-24	-49	-57	-32	-89	-125

Note: the table shows the sum of the impacts of the ECB policies introduced in June 2014, January 2015, March 2018, September 2019, and June 2020. We do not include the estimated impact from the introduction of the PEPP in March 2020, as for Italy, this impact may have been contaminated with volatile markets assessments due to the proximity to the outbreak of the Covid-19 pandemic. The sum is calculated for each country, maturity, and evaluation horizon. Impacts are traced in time from the month prior to the given announcement till 6 months after. The results are shown in basis points. The y^{drf}-curve captures country specific nominal growth and credit, liquidity, and redenomination risk. TP refers to "term premium".

Table 4: Impact summary in basis points

4 Conclusions

We contribute to the literature by deriving a term structure model that is parameterised by factors with direct interpretations as: the short rate, a measure that encompasses the time-varying nominal growth of the economy and issuer specific risks, and the slope and curvature of the term structure of term premia. The model falls in the family of no-dominance framework developed by Feunou, Fontaine, Le, and Lundblad (2021).

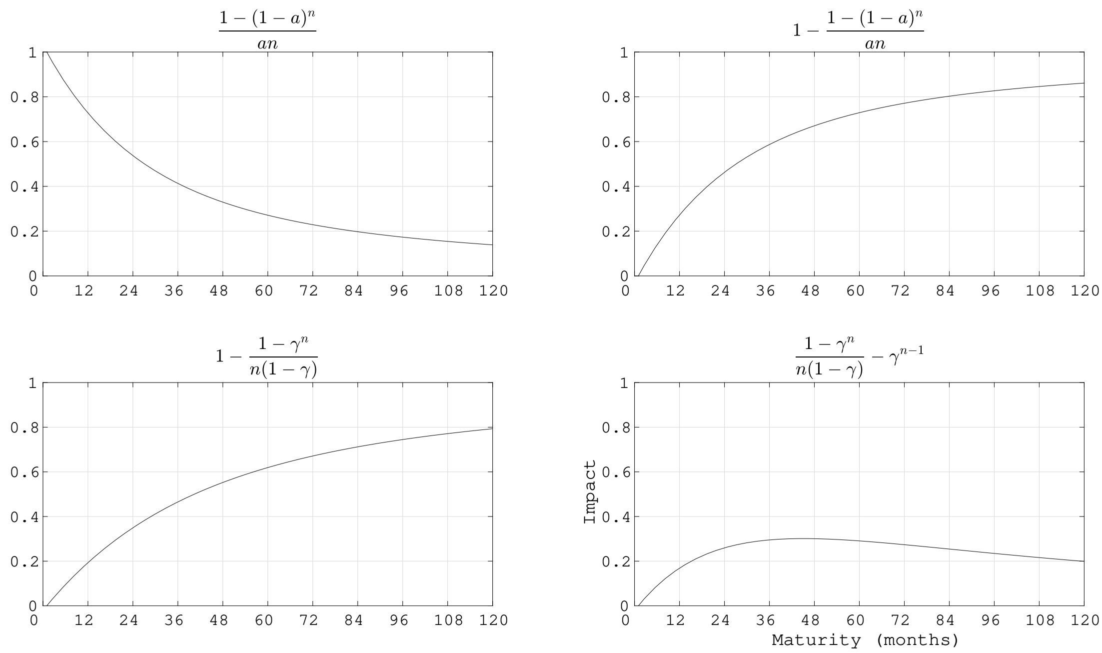
Our model is flexible and lends itself to investment and policy analysis, as it relies on factors that are impacted by central bank policy and that are used by investment professional when assessing investment opportunities: the short rate reflects conventional monetary policy and its future path is impacted by forward guidance; unconventional policies, such as asset purchases, impact sovereign credit, liquidity, and redenomination risk, as well as the term structure of term premia.

In an empirical application of the model we integrate the amount of free-floating government bonds, i.e. outstanding debt less holdings by the central bank and foreign official institutions, as an unspanned factor. This version of the model is used to assess the impact of ECB policies on the German and Italian yield curves. Six ECB policy announcements are evaluated from June 2014 to June 2020. The evaluation separates the policy impact between extraction of credit, liquidity, and redenomination risk, on the one hand, and duration risk, on the other hand.

We find that ECB policies have extracted both credit, liquidity, and redenomination as well as duration risk from the Italian market. Across the evaluated policy initiatives approximately 1/3 of the yield compression is obtained via credit, liquidity, and redenomination risk, while 2/3 of the compression stems from term premia compression, i.e. duration risk. In the German market the ECB policies have exclusively extrated duration risk. It appears that the basis point amount of duration risk extraction is more or less the same in the Germany and Italian markets, when seen across the sample of policy dates. Duration induced compressions in the German market amounts to 25, 85, and 125 basis points, at the 1Y, 5Y and 10Y maturities, respectively. The corresponding estimates for duration induced compressions in the Italian market are 30, 90, and 125 basis points. In addition, credit, liquidity, and redenomination risk has been extracted from the Italian market of approximately 25, 50, and 57 basis points. No credit, liquidity, and redenomination risk was extracted from the German market.

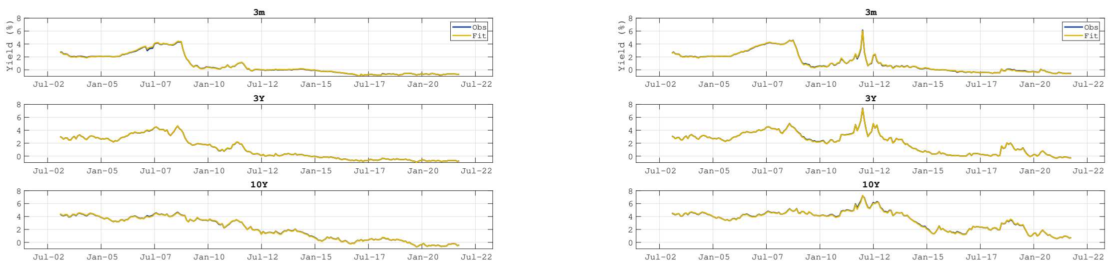
Finally it is found that credit, liquidity, and redenomination risk is mainly extracted from short to medium term maturities, while duration risk is predominantly extracted from medium to long term maturities.

Figures



Note: This figure shows the yield curve factor loadings derived in (12) across maturities from 0 to 120 month. The upper-left panel shows the first element in b_n, the upper-right panle the second element, the lower-left the third element, and the last element of b_n is shown in the lower-right panel. The x-axis in the plotted panels shows the maturity in months and the y-axis shows the normalised impact of the respective factors on the yield curve.

Figure 1: The factor loadings

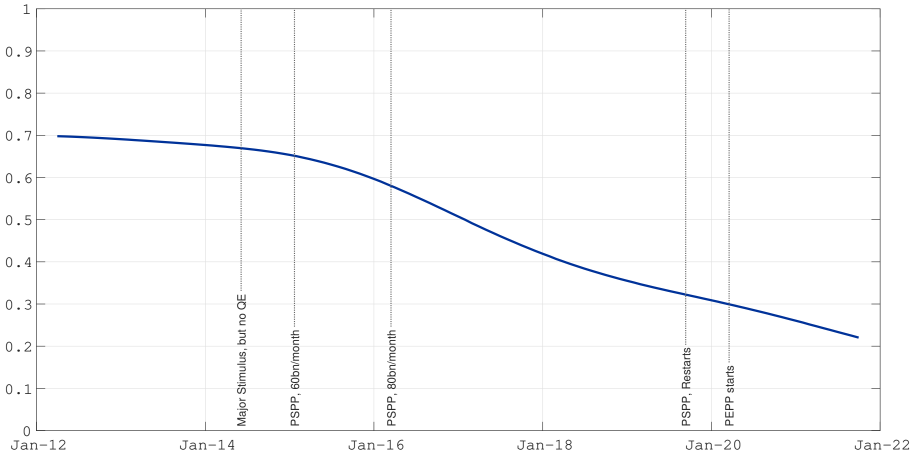


(a) Germany

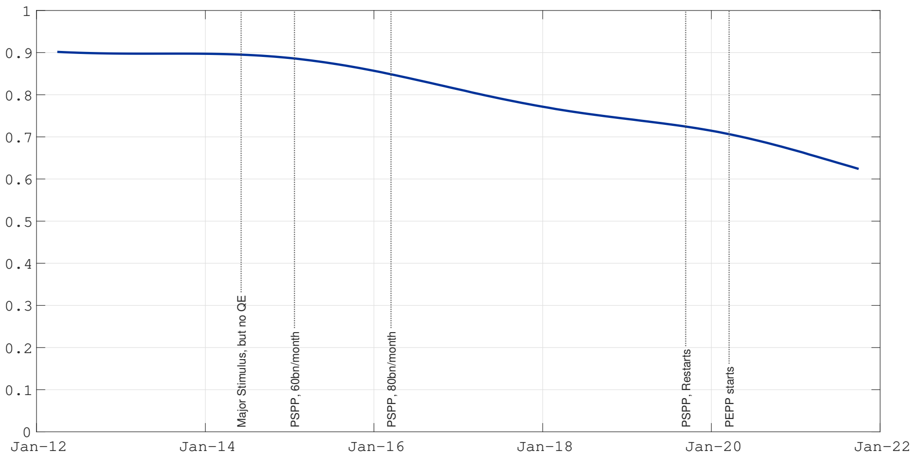
(b) Italy

Note: This figure shows observed and fitted yields for the German and Italian markets. A selected set of maturities are displayed.

Figure 2: Observed and fitted yields



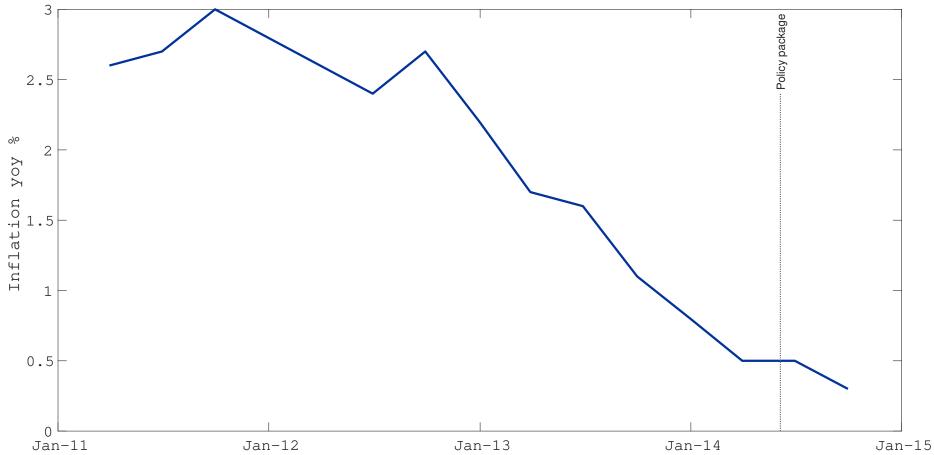
(a) Germany



(b) Italy

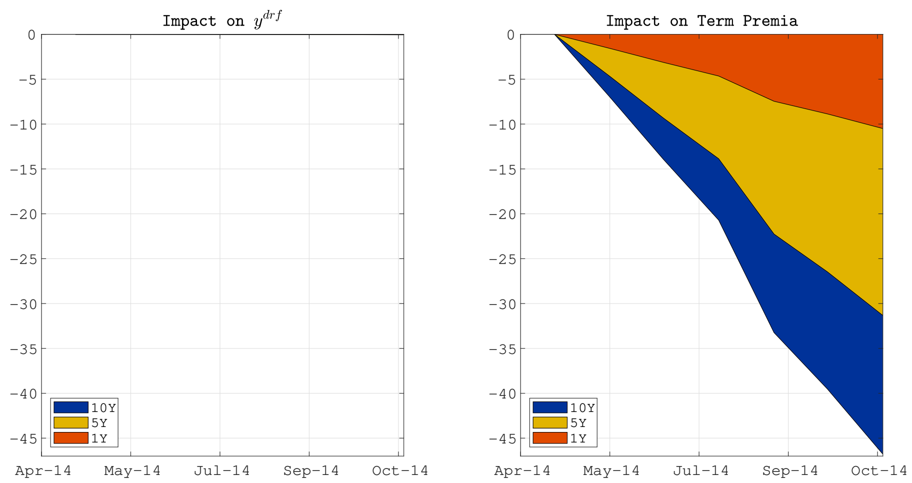
Note: The figure shows the evolution of the free float variable (FF) calculated for Germany and Italy over the period from January 2003 to September 2021 using IMF data, see Arslanalp and Tsuda (2012). Using the relevant data sources the free float variable is calculated as shown in (24): $FF = \texttt{Gov. debt} - \texttt{holdings of foreign official inst.} - \texttt{Holdings of domestic CB}/\texttt{Gov. debt}$. Relevant policy dates are displayed. These are the dates for which the term structure impacts are calculated. In June 2014 the ECB announced measures to enhance the functioning of the monetary policy transmission mechanism by supporting lending to the real economy comprising via a series of targeted longer-term refinancing operations; in January 2015 the Public Sector Purchase Programme (PSPP) was announced with an initial purchase volume of Eur 60 billion per month; In March 2016 the PSPP volume was increased to Eur 80 billion per month; After ending in 2018 PSPP purchases were resumed in September 2019; and in March 2020 the Pandemic Emergency Purchase Programme (PEPP) was initiated, with a programme extension in June where the envelope was nearly doubled in size from Eur 700 billion to 1,300 billion.

Figure 3: Free floats and policy dates

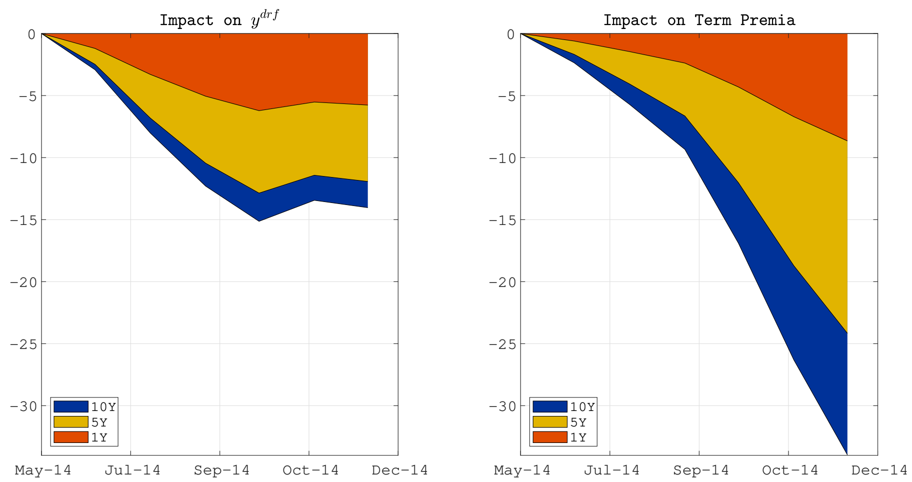


Note: The figure shows eurozone flash estimate of the harmonised index of consumer prices (HICP).

Figure 4: Euro zone inflation from 2011 to 2014



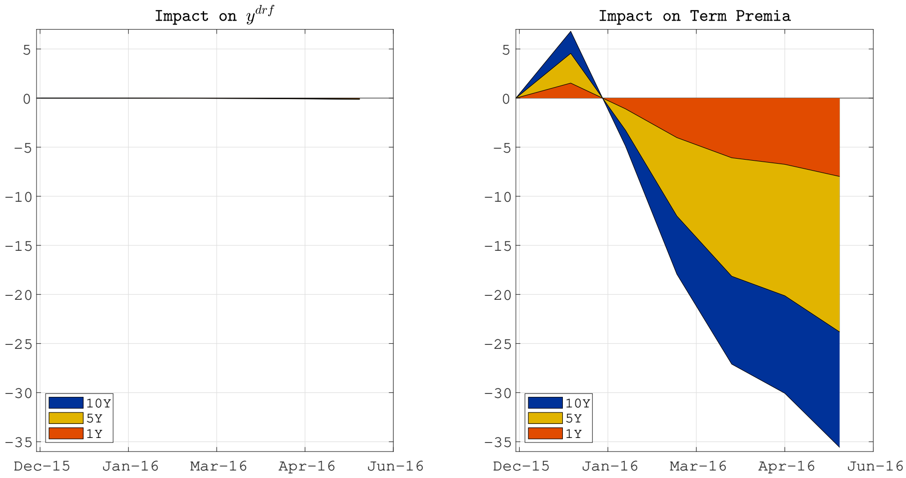
(a) Germany



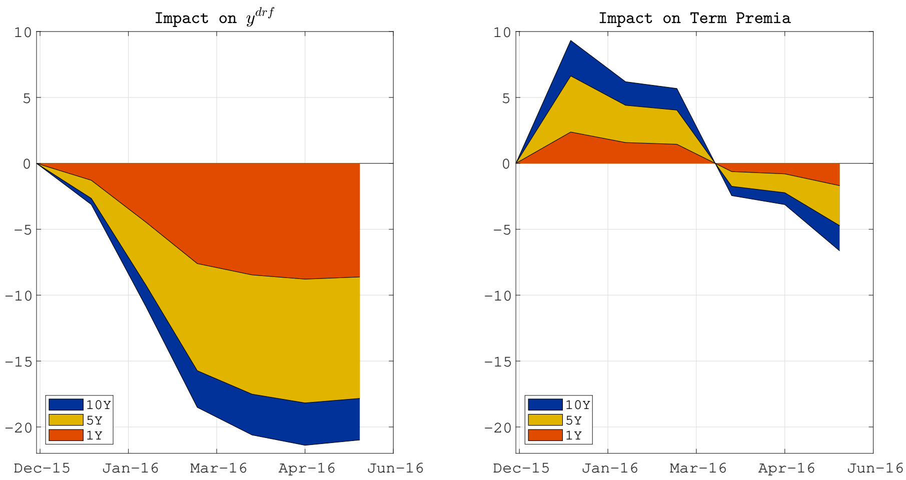
(b) Italy

Note: the figure shows the impacts of the measures announced on 5 June 2014. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the extension of the programme in June till 6 months after. The results are shown in basis points.

Figure 5: Policy impact June 2014



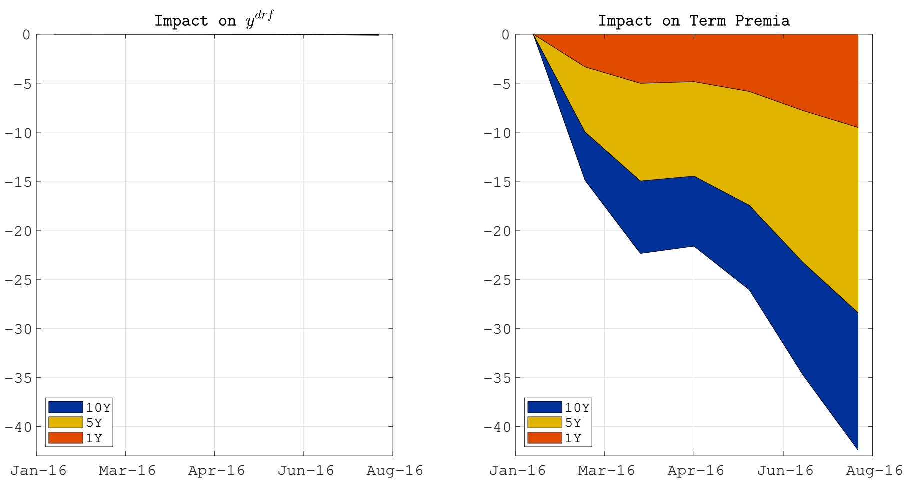
(a) Germany



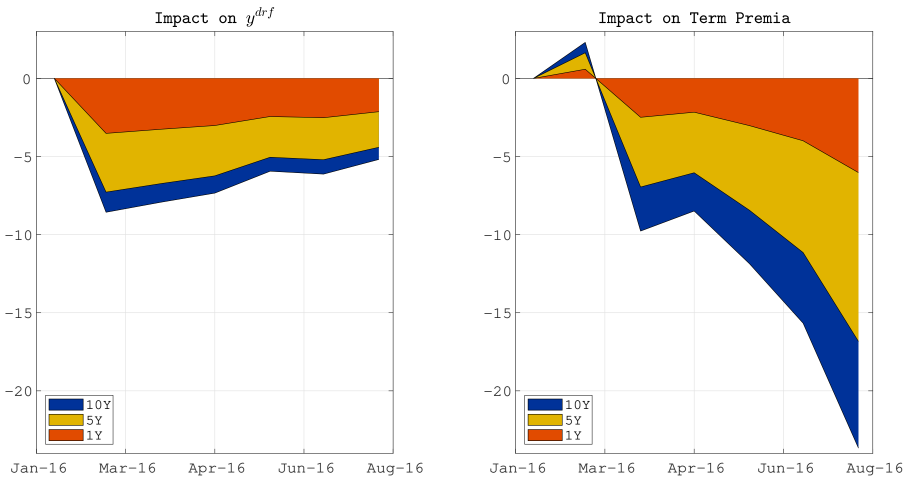
(b) Italy

Note: the figure shows the impacts of the measures announced on 22 January 2015. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the extension of the programme in June till 6 months after. The results are shown in basis points.

Figure 6: Policy impact Jan 2015



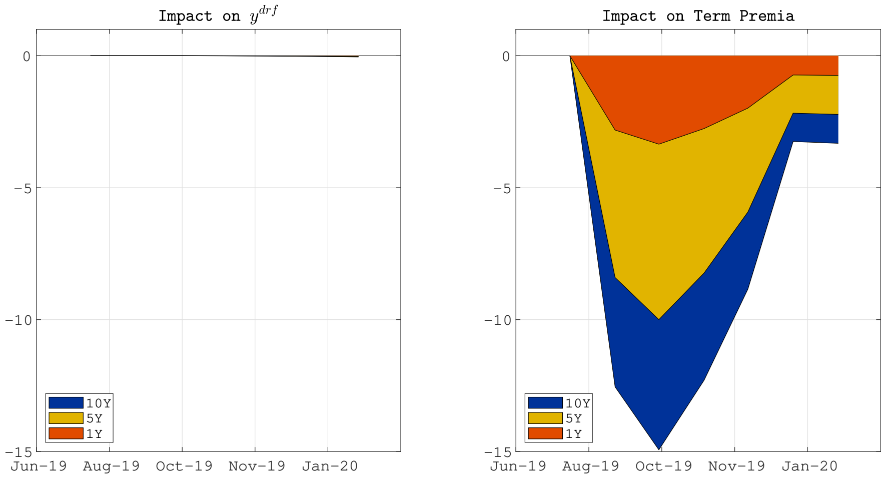
(a) Germany



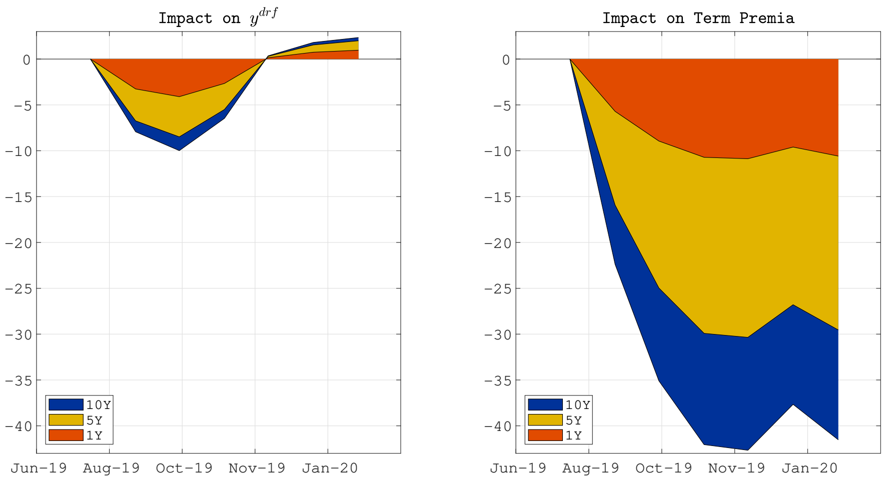
(b) Italy

Note: the figure shows the impacts of the measures announced on 15 March 2016. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the extension of the programme in June till 6 months after. The results are shown in basis points.

Figure 7: Policy impact March 2016



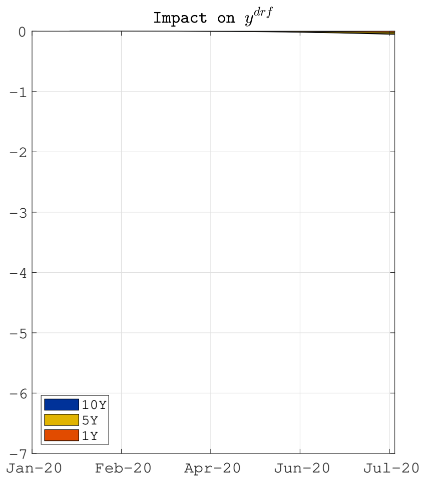
(a) Germany



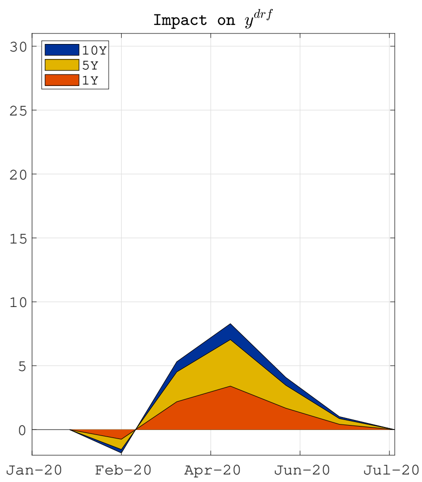
(b) Italy

Note: the figure shows the impacts of the measures announced on 12 September 2019. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the extension of the programme in June till 6 months after. The results are shown in basis points.

Figure 8: Policy impact September 2019



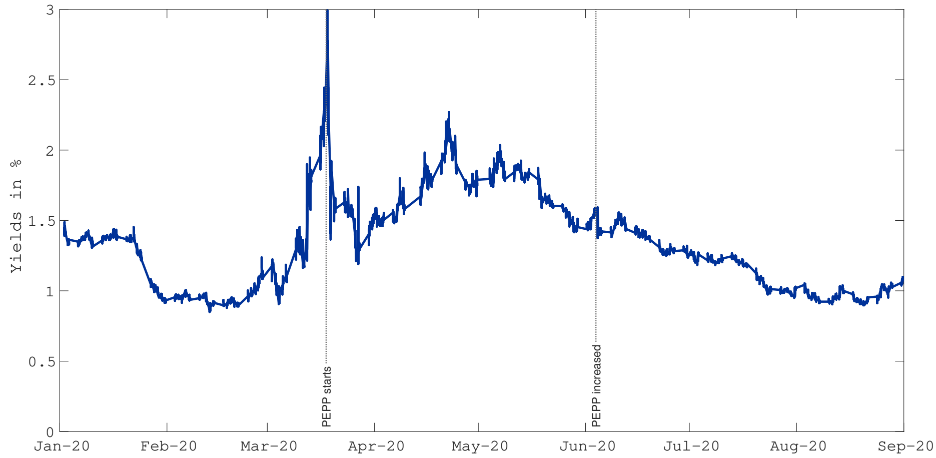
(a) Germany



(b) Italy

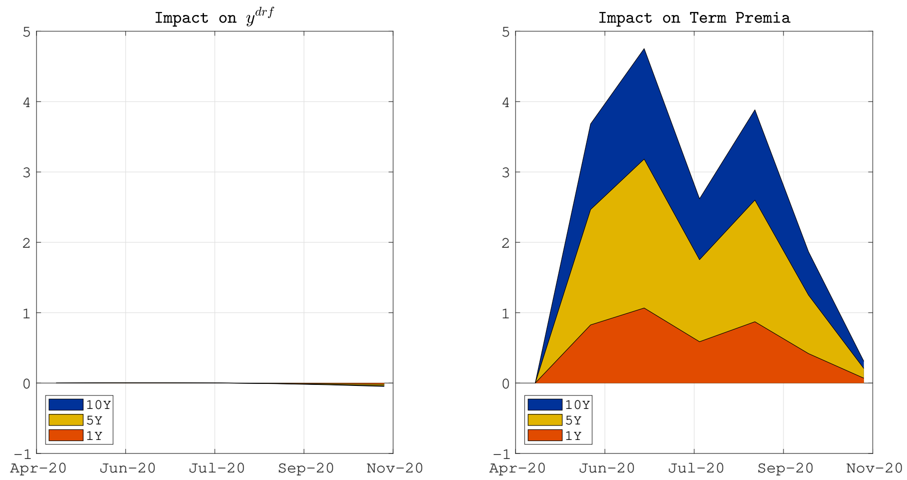
Note: the figure shows the impacts of the pandemic emergency purchase programme announced by the ECB on 18 March 2020. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the announcement of the programme till 6 months after. The results are shown in basis points.

Figure 9: Policy impact March 2020

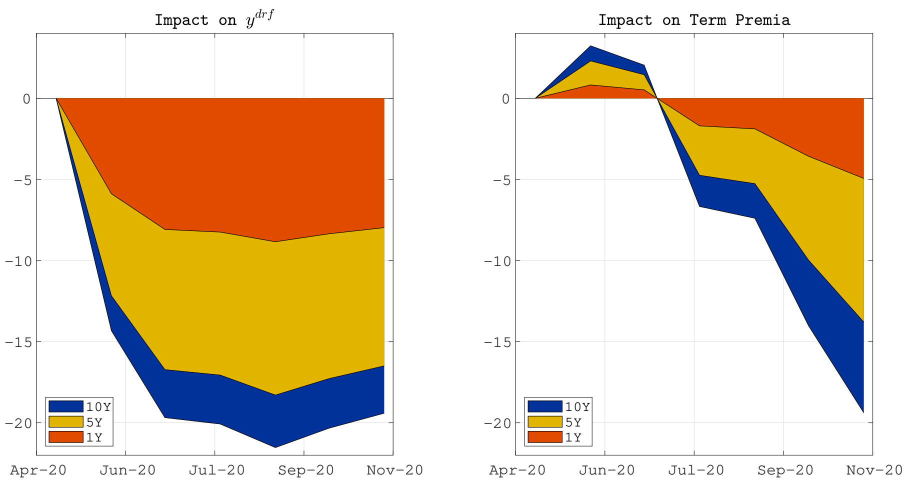


Note: the figure shows the intradaily evolution of the Italian 10-year yield during January to September 2020.

Figure 10: Italian 10-year yields during 2020



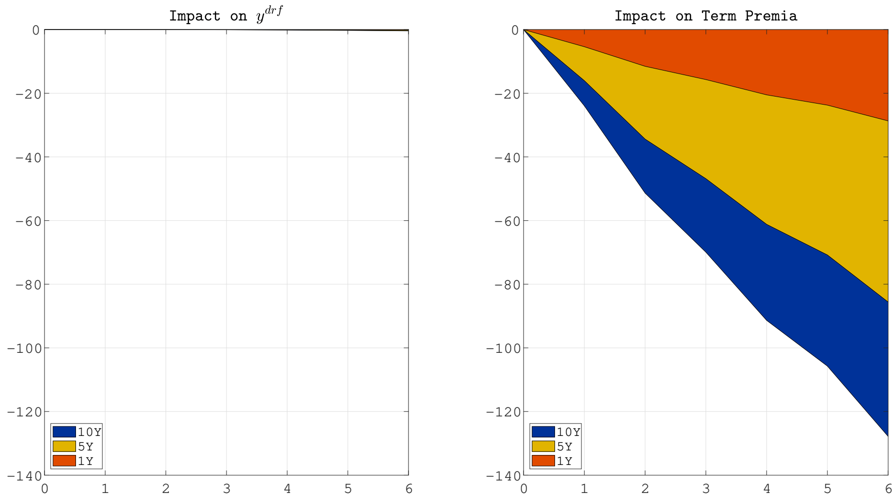
(a) Germany



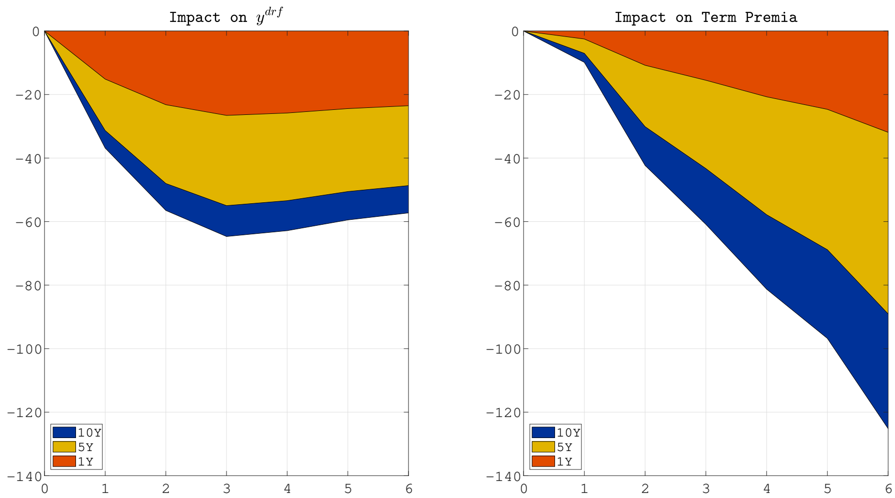
(b) Italy

Note: the figure shows the impacts of the increased envelope of the pandemic emergency purchase programme on 4 June 2020. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the extension of the programme in June till 6 months after. The results are shown in basis points.

Figure 11: Policy impact June 2020



(a) Germany



(b) Italy

Note: the figure shows the sum of the impacts of the ECB policies introduced in June 2014, January 2015, March 2018, September 2019, and June 2020. We do not include the estimated impact from the introduction of the PEPP in March 2020, as for Italy, this impact may have been contaminated with volatile markets assessments due to the proximity to the outbreak of the Covid-19 pandemic. The sum is calculated for each country, maturity, and evaluation horizon. Impacts are shown on the German (panel a) and Italian (panel b) yield curves at 1Y, 5Y, and 10Y maturities, and are traced in time from the month prior to the given announcement till 6 months after. The results are shown in basis points.

Figure 12: Summary of the impacts

References

Ang, A., and M. Piazzesi (2003): "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables", *Journal of Monetary Economics*, 50(4), 745–787.

Arslanalp, S., and T. Tsuda (2012): "Tracking Global Demand for Advanced Economy Sovereign Debt", IMF working paper, No 12/284.

Bjorheim, J., J. Coche, A. Joia, and V. Sahakyan (2018): "A macro-based process for actively managing sovereign bond exposures", in *Advances in the practice of public investment management*, ed. by N. Bulusu, J. Coche, A. Rivadeneyra, V. Sahakyan, and G. Yanou. Palgrave MacMillan.

Christensen, J. H. E., F. X. Diebold, and G. D. Rudebusch (2011): "The affine arbitrage-free class of Nelson-Siegel term structure models", *Journal of Econometrics*, 164, 4–20.

Collin-Dufresne, P., R. S. Goldstein, and C. S. Jones (2008): "Identification of Maximal Affine Term Structure Models", *Journal of Finance*, 63(2), 743–795.

Coroneo, L., K. Nyholm, and R. Vidova-Koleva (2008): "How arbitragefree is the Nelson-Siegel model?", ECB Working Paper no 874.

Costain, J., G. Nuño, and C. Thomas (2021): "The Term Structure of Interest Rates in a Heterogeneous Monetary Union", Unpublished Manuscript.

Cox, J., J. Ingersoll, and S. Ross (1985): "A Theory of the Term Structure of Interest Rates", *Econometrica*, 53, 385–407.

Dai, Q., and K. Singleton (2000): "Specification analysis of affine term structure models", *Journal of Finance*, 55, 1943–1978.

Diebold, F. X., and C. Li (2006): "Forecasting the Term Structure of Government Bond Yields", *Journal of Econometrics*, 130, 337–364.

Diebold, F. X., and G. D. Rudebusch (2013): *Yield Curve Modeling and Forecasting: The Dynamic Nelson-Siegel Approach*. Princeton University Press, Princeton, New Jersey, USA.

Diebold, F. X., G. D. Rudebusch, and S. B. Aruoba (2006): "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach", *Journal of Econometrics*, 131, 309–338.

Duffie, D., and R. Kan (1996): "A yield-factor model of interest rates", *Mathematical Finance*, 6, 379–406.

Eser, F., W. Lemke, K. Nyholm, S. Radde, and A. L. Vladu (2019): "Tracing the impact of the ECB's asset purchase programme on the yield curve", ECB working paper, No 2293.

Feunou, B., J.-S. Fontaine, A. Le, and C. Lundblad (2021): "Tractable Term-Structure Models", Management Science, conditional acceptance.

Holston, K., T. Laubach, and J. C. Williams (2017): "Measuring the Natural rate of Interest: International Trends and Determinants", *Journal of International Economics*, 108, 59–75.

Joslin, S., K. J. Singleton, and H. Zhu (2011): "A New Perspective on Gaussian Dynamic Term Structure Models", *Review of Financial Studies*, 24, 926–970.

Joyce, M., A. Lasaosa, I. Stevens, and M. Tong (2011): "The Financial Market Impact of Quantitative Easing in the United Kingdom", *International Journal of Central Banking*, 7, 113–161.

Joyce, M., D. Miles, A. Scott, and D. Vayanos (2012): "Quantitative Easing and Unconventional Monetary Policy - An Introduction", *The Economic Journal*, 122, 271–288.

Krippner, L. (2013): "Measuring the stance of monetary policy in zero lower bound environments", *Economics Letters*, 118(1), 135–138.

Li, C., and M. Wei (2013): "Term structure modeling with supply factors and the Federal Reserve's large-scale asset purchase programs", *International Journal of Central Banking*, 9, 3–39.

Moody's (2010): "Modeling Methodology from Moody's KMV: CDS-implied EDF™ Credit Measures and Fair-value Spreads", Whitepaper, March, Moody's Analytics.

Nelson, C., and A. Siegel (1987): "Parsimonious modeling of yield curves", *Journal of Business*, 60, 473–89.

Piazzesi, M. (2010): "Affine term structure models", in *Handbook of Financial Econometrics*, ed. by Y. Ait-Sahalia, and L. P. Hansen. North Holland, Elsevier.

Roberts, J. M. (2018): "An Estimate of the Long-Term Neutral Rate of Interest", https://www.federalreserve.gov/econres/notes/feds-notes/estimate-of-the-long-term-neutral-rate-of-interest-20180905.htm.

Vasicek, O. (1977): "An equilibrium characterization of the term structure", *Journal of Financial Economics*, 5, 177–188.

Vayanos, V., and J. Vila (2009): "A Preferred-Habitat Model of the Term structure of Interest Rates", NBER Working Paper.

A Appendix: An arbitrage free version of the model

Our purpose here is to illustrate how the standard linear modelling set-up (see, e.g., Duffie and Kan (1996), Dai and Singleton (2000), and Ang and Piazzesi (2003)) can be used to derive a discrete-time arbitrage-free version of the model presented in the text.

Within the continuous-time setting Christensen, Diebold, and Rudebusch (2011) have shown how to maintain the parametric loading structure of the Nelson and Siegel (1987), while ensuring that arbitrage constraints are fulfilled.[13]

As before, let X_t denote the vector of the modelled yield curve factors, at time t. Furthermore, let the dynamics of X_t be governed by vector autoregressive (VAR) processes of order one, under both the empirical measure, $\mathbb{P}$, and the pricing measure, $\mathbb{Q}$:

$$X_t = k^{\mathbb{P}} + \Phi^{\mathbb{P}} \cdot X_{t-1} + \Sigma^{\mathbb{P}} \epsilon_t^{\mathbb{P}}, \qquad \epsilon_t^{\mathbb{P}} \sim N(0,1) \tag{30}$$

$$X_t = k^{\mathbb{Q}} + \Phi^{\mathbb{Q}} \cdot X_{t-1} + \Sigma^{\mathbb{Q}} \epsilon_t^{\mathbb{Q}}, \qquad \epsilon_t^{\mathbb{Q}} \sim N(0,1). \tag{31}$$

with $\Sigma\Sigma' = \Omega$ being the variance of the residuals, and it is assumed that $\Sigma^{\mathbb{P}} = \Sigma^{\mathbb{Q}}$. To define our mode we use:

$$\Phi^{\mathbb{Q}} = \Phi = \begin{bmatrix} 1-a & a & 1-\gamma & 1-\gamma \\ 0 & 1 & 1-\gamma & 1-\gamma \\ 0 & 0 & \gamma & \gamma-1 \\ 0 & 0 & 0 & \gamma \end{bmatrix} \tag{32}$$

As the first element in X_t is defined to be the one-period short rate, we have:

$$r_t = \rho_0 + \rho_1' X_t. \tag{33}$$

with the following constraints on (33): $\rho_0 = 0$ and $\rho_1 = [1, 0, 0, 0]'$.

We now impose absence of arbitrage on the model by introducing the unique pricing mechanism, that governs all traded assets:

$$P_{t,\tau} = \mathbb{E}_t \left[M_{t+1} \cdot P_{t+1,\tau-1} \right] \tag{34}$$

The idea here is that when the bond matures at time T, its value is know with certainty, since it is default-free: the bond pays its principal value on that day, so $P_{T,0} = 1$. At any time $t+j$ before maturity, the price of the bond can therefore be found as the one-period discounted-value of the price at time $t+j+1$, all the way back to time t. Discounting is done using the stochastic discount factor (also called the pricing kernel), which is denoted by M_t, and this quantity is assumed to be given by:

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t' \epsilon_{t+1}^{\mathbb{P}} \right) \tag{35}$$

with

$$\lambda_t = \lambda_0 + \lambda_1 \cdot X_t, \tag{36}$$

where λ_t is of dimension (4×1) in our application, because we have four factors, λ_0 is of dimension (4×1), and λ_1 is a matrix of dimension (4×4).

It is recalled that:

$$y_{t,\tau} = -\frac{1}{\tau} \log(P_{t,\tau}), \tag{37}$$

and that we can write the yield curve expression as an affine function:

$$y_{t,\tau} = -\frac{A_\tau}{\tau} - \frac{B_\tau'}{\tau} X_t. \tag{38}$$

The bond price is therefore exponential affine in terms of A_τ and B_τ:

$$P_{t,\tau} = \exp\left(A_\tau + B_\tau' X_t \right). \tag{39}$$

[13] See also, Krippner (2013) and Diebold and Rudebusch (2013).

To derive closed-form expressions for A_τ and B_τ, the fundamental pricing equation is invoked (35):

$$P_{t,\tau} = \mathbb{E}_t\left[M_{t+1}\cdot P_{t+1,\tau-1}\right] \tag{40}$$
$$= \mathbb{E}_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}^{\mathbb{P}}\right)\cdot\exp\left(A_{\tau-1} + B_{\tau-1}'X_{t+1}\right)\right]. \tag{41}$$

The expression for X_{t+1} (see equation 30) is substituted:

$$P_{t,\tau} = \mathbb{E}_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}^{\mathbb{P}}\right)\cdot\exp\left(A_{\tau-1} + B_{\tau-1}'\left(k^{\mathbb{P}} + \Phi^{\mathbb{P}}X_t + \Sigma\epsilon_{t+1}^{\mathbb{P}}\right)\right)\right], \tag{42}$$

and, the terms are then separated into two groups: one to which the expectations operator should be applied, i.e. $t+1$ terms, and another group, which are known at time t:

$$\begin{aligned} P_{t,\tau} =& \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t + A_{\tau-1} + B_{\tau-1}'k^{\mathbb{P}} + B_{\tau-1}'\Phi^{\mathbb{P}}X_t\right) \\ &\cdot\mathbb{E}_t\left[\exp\left(-\lambda_t'\epsilon_{t+1}^{\mathbb{P}} + B_{\tau-1}'\Sigma\epsilon_{t+1}^{\mathbb{P}}\right)\right]. \end{aligned} \tag{43}$$

The question is then, how can we calculate the expectations part of (43):

$$\mathbb{E}_t\left[\exp\left(-\lambda_t' + B_{\tau-1}'\Sigma\right)\epsilon_{t+1}^{\mathbb{P}}\right]. \tag{44}$$

To this end, the moment generating function of the multivariate normal distribution is used. Since $\epsilon^{\mathbb{P}} \sim N(0, I)$, it is known that:

$$\mathbb{E}[\exp(a'\epsilon^{\mathbb{P}})] = \exp\left(\frac{1}{2}a'\cdot I\cdot a\right), \tag{45}$$

so, the expectation in (43) can be calculated, using $a' = (-\lambda_t' + B_{\tau-1}'\Sigma)$, as:

$$\begin{aligned} &\exp\left[\frac{1}{2}(-\lambda_t' + B_{\tau-1}'\Sigma)\cdot I\cdot(-\lambda_t' + B_{\tau-1}'\Sigma)'\right] \\ =&\exp\left[\frac{1}{2}(-\lambda_t' + B_{\tau-1}'\Sigma)\cdot I\cdot(-\lambda_t + \Sigma'B_{\tau-1})\right] \\ =&\exp\left[\frac{1}{2}\left(\lambda_t'\lambda_t - \lambda_t'\Sigma'B_{\tau-1} - B_{\tau-1}'\Sigma\lambda_t + B_{\tau-1}'\Sigma\Sigma'B_{\tau-1}\right)\right], \end{aligned} \tag{46}$$

and, since $B_{\tau-1}'\Sigma\lambda_t$ is a scalar, and for a scalar h, we know that $h = h'$, so $B_{\tau-1}'\Sigma\lambda_t = \lambda_t'\Sigma'B_{\tau-1}$. We can then write:

$$\begin{aligned} &\mathbb{E}_t\left[\exp\left(-\lambda_t' + B_{\tau-1}'\Sigma\right)\epsilon_{t+1}^{\mathbb{P}}\right] \\ =&\exp\left[\left(\frac{1}{2}\lambda_t'\lambda_t - B_{\tau-1}'\Sigma\lambda_t + \frac{1}{2}B_{\tau-1}'\Sigma\Sigma'B_{\tau-1}'\right)\right]. \end{aligned} \tag{47}$$

This term is then reinserted into (43), giving:

$$P_{t,\tau} = \exp\left(-r_t + A_{\tau-1} + B_{\tau-1}'k^{\mathbb{P}} + B_{\tau-1}'\Phi^{\mathbb{P}}X_t - B_{\tau-1}'\Sigma\lambda_t + \frac{1}{2}B_{\tau-1}'\Sigma\Sigma'B_{\tau-1}'\right). \tag{48}$$

It is recalled that $r_t = \rho_1'X_t$, and that $\lambda_t = \lambda_0 + \lambda_1X_t$. Inserting these expressions into (48), gives:

$$\begin{aligned} P_{t,\tau} = \exp\Big(&-\rho_1'X_t + A_{\tau-1} + B_{\tau-1}'k^{\mathbb{P}} + B_{\tau-1}'\Phi^{\mathbb{P}}X_t \\ &-B_{\tau-1}'\Sigma\left(\lambda_0 + \lambda_1X_t\right) + \frac{1}{2}B_{n-1}'\Sigma\Sigma'B_{\tau-1}'\Big). \end{aligned} \tag{49}$$

Reorganising this expression into terms that load on X_t and terms that do not, help matching coefficients with respect to equation (39):

$$\begin{aligned} P_{t,\tau} = \exp\Big(&A_{\tau-1} + B_{\tau-1}'\left(k^{\mathbb{P}} - \Sigma\lambda_0\right) + \frac{1}{2}B_{\tau-1}'\Sigma\Sigma'B_{\tau-1}' \\ &+B_{\tau-1}'\Phi^{\mathbb{P}}X_t - \rho_1'X_t - B_{\tau-1}'\Sigma\lambda_1X_t\Big), \end{aligned} \tag{50}$$

which is:

$$P_{t,\tau} = \exp\Bigg(A_{\tau-1} + B'_{\tau-1}\left(k^{\mathbb{P}} - \Sigma\lambda_0\right) + \frac{1}{2}B'_{\tau-1}\Sigma\Sigma' B'_{\tau-1} \\ + \left[B'_{\tau-1}\left(\Phi^{\mathbb{P}} - \Sigma\lambda_1\right) - \rho'_1\right] X_t \Bigg). \tag{51}$$

Matching the coefficients of (51) with those of (39) establishes the recursive formulas for A_n and B_n:

$$A_n = A_{n-1} + B'_{n-1}k^{\mathbb{Q}} + \frac{1}{2}B'_{n-1}\Sigma\Sigma' B'_{n-1} \tag{52}$$

$$B'_n = B'_{n-1}\Phi^{\mathbb{Q}} - \rho'_1 \tag{53}$$

with $k^{\mathbb{Q}} = k^{\mathbb{P}} - \Sigma\lambda_0$, and $\Phi^{\mathbb{Q}} = \Phi^{\mathbb{P}} - \Sigma\lambda_1$. Recall that $\rho_0 = 0$ in our model setup. Using recursive substitution, we realise that the expression for B'_n also can be written in the following way:[14]

$$B_n = -\left[\sum_{k=0}^{\tau-1}\left(\Phi^{\mathbb{Q}}\right)^k\right]' \cdot \rho_1. \tag{54}$$

which is the same expression as we obtain in the text for the loading structure under the no-dominance requirement. Hence the arbitrage-free version of the model presented in the text is obtained by adding a constant A_n/n to the yield equation of the model:

$$A_n = A_{n-1} + B'_{n-1}k^{\mathbb{Q}} + \frac{1}{2}B'_{n-1}\Sigma\Sigma' B'_{n-1}. \tag{55}$$

[14]We see this by the use of an example. For $\tau = 3$, we have:

$$\begin{aligned}
B'_1 &= -\rho'_1 \\
B'_2 &= B'_1\Phi^{\mathbb{Q}} - \rho'_1 = -\rho'_1\Phi^{\mathbb{Q}} - \rho'_1 \\
B'_3 &= B'_2\Phi^{\mathbb{Q}} - \rho'_1 = (-\rho'_1\Phi^{\mathbb{Q}} - \rho'_1)\Phi^{\mathbb{Q}} - \rho'_1 \\
&= -\rho'_1\left(\Phi^{\mathbb{Q}}\right)^2 - \rho'_1\Phi^{\mathbb{Q}} - \rho'_1 \\
&= -\rho'_1\left(\left(\Phi^{\mathbb{Q}}\right)^2 + \left(\Phi^{\mathbb{Q}}\right)^1 + \left(\Phi^{\mathbb{Q}}\right)^0\right) \\
&= -\rho'_1\left[\sum_{k=0}^{2}\left(\Phi^{\mathbb{Q}}\right)^k\right]
\end{aligned}$$

so,

$$B_3 = -\left[\sum_{k=0}^{2}\left(\Phi^{\mathbb{Q}}\right)^k\right]' \rho_1,$$

which generalises to equation (54).

B Appendix: Estimating the monthly output gap series

We use quarterly and annual consensus forecasts on real economic activity and monthly industrial production data, collected from the national statistical offices and IMF, to derive monthly output gaps. A mixed-frequency state-space model, following and expanded version of Bjorheim, Coche, Joia, and Sahakyan (2018), is applied with the following state-equation:

$$\begin{bmatrix} \ln GDP_t \\ \ln IP_t \\ xGap_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ \beta_1 & \beta_1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \ln GDP_t \\ \ln GDP_t^{pot} \\ \ln GDP_{t-1}^{pot} \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_t \\ 0 \end{bmatrix} \tag{56}$$

where GDP and IP (industrial production) are in levels and $xGap$ is in percentage. The following transition-equation completes the model:

$$\begin{bmatrix} \ln GDP_t \\ \ln GDP_t^{pot} \\ \ln GDP_{t-1}^{pot} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \ln GDP_t^{cyc} \\ \ln GDP_t^{pot} \\ \ln GDP_{t-1}^{pot} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ 0 \end{bmatrix} \tag{57}$$

with potential and cyclical components of GDP treated as latent variables.